LOCALIZATION AND THE VACUUM

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1. INTRODUCTION

In a recent paper Malament (1992) has proved some very elegant theorems concerning the detection of particles in the vacuum state of a relativistic quantum field theory. Firstly he shows that there is a nonvanishing probability that a localized particle detector of a most general sort will 'fire' in response to its coupling to an initial vacuum state of the field. In a second theorem he shows the existence of correlations between the 'firing' of a localized detector and any other local observable in the field, irrespective of the separation of the two localizations in question. The object of the present paper is to investigate the significance of these theorems in the general context of understanding and interpreting relativistic quantum field theory.

2. THE VACUUM OF A RELATIVISTIC QUANTUM FIELD

In a sense Malament is directing attention to well-known, if somewhat paradoxical features of relativistic quantum field theory (RQFT). Malament obtains his result by using the famous Reeh-Schlieder theorem[1] in the form that the vacuum is cyclic for the whole Hilbert space of the field with respect to any local algebra associated with an arbitrary bounded open set in Minkowski spacetime. The intuitive idea here is that acting on the vacuum with all the members of the local algebra R(0) attached to the open set 0 would get us as close as we like to any state of the field.
This seems amazing since it seems to say that performing operations in some region 0 could generate excitations in the field that were localized in some region 0′ that did not overlap with 0, and was spacelike separated from 0 by an arbitrarily large interval. But how could this happen in a local field theory? The answer is of course that the vacuum is a highly nonlocal state of the field. Intuitively, tweaking the vacuum over here can produce a response over there, not by action-at-a-distance but by exploiting the correlations built into the relativistic vacuum between distant events. Essentially Malament's second theorem is the vital clue to what is going on in ~~his first theorem~~. the Reeh-Schlieder theorem

However, it is important to realise that these vacuum

correlations are not independent of distance, as in Bell-type correlations, but fall off exponentially with distance on a scale set by the Compton wavelength of a massive field, or the ordinary wavelength of a photon field.[2] It is well-known that the correlations aximally violate the Bell inequality, i.e., achieving the so-called Cirelson bound of $2\sqrt{2}$ against the classical limit of 2 for the Bell inequalities[3]. Malament is quite right to say that the correlations are nonvanishing for any distance separating the localizations, but the exponential form of the distance dependence is of of vital importance in assessing whether vacuum correlations could be used to perform a source-free variant of the Bell experiment.

But let us turn now directly to the first theorem. Malament presents this as an exercise in measurement theory. I shall give a different sort of proof of essentially the same result that does not discuss detectors at all.

Reconstruction of Malament Theorem. 1.

What can be measured *locally* is in 1:1 correspondence with its projection operators in the local algebra.

Note that $\forall A(O)$, $P_{A(O)\Omega} \notin R(O)$

so it is never a local question to ask, are we in state $A(O)\Omega$?

Consider $P_O \in R(O)$

Von measurements evaluate.

$$p = \mathrm{Prob}^{\Omega}(P_O = 1)$$

$$= \|P_O\Omega\|^2$$

$P \in R(O)$

$P\Omega \neq 0$

$\Updownarrow$

$P_\Omega = \langle \Omega, P\Omega \rangle \neq 0$

$$p = 0 \Rightarrow P_O\Omega = 0$$

$$\Rightarrow P_O = 0 \quad \text{by R-S Theorem}$$

$$\therefore P_O \neq 0 \Rightarrow \mathrm{Prob}^{\Omega}(P_O = 1) \neq 0$$

So any non-trivial local question has nonvanishing probability of answer yes

P.T.O

Michael,

One also has the following assertion which has the flavor of my 2nd proposition

<u>Prop</u>. Given any two spacelike related open sets O_1 and O_2, there exist projection operators $P_1 \in R(O_1)$ and $P_2 \in R(O_2)$ s.t.

$$\langle \Omega, P_1 P_2 \Omega \rangle \neq \langle \Omega, P_1 \Omega \rangle \langle \Omega, P_2 \Omega \rangle .$$

<u>Pf</u>. Consider <u>any</u> projection operator $P_1 \in R(O_1)$. Assume that

$$\langle \Omega, P_1 P_2 \Omega \rangle = \langle \Omega, P_1 \Omega \rangle \langle \Omega, P_2 \Omega \rangle$$

for <u>all</u> projection operators $P_2 \in R(O_2)$. Let

$$\hat{P}_1 = P_1 - \langle \Omega, P_1 \Omega \rangle I .$$

Then

$$\langle \Omega, \hat{P}_1 P_2 \Omega \rangle = 0$$

for all $P_2 \in R(O_2)$. So

$$\langle \hat{P}_1 \Omega, P_2 \Omega \rangle = 0$$

for all $P_2 \in R(O_2)$. But if this holds for all projection operators in $R(O_2)$, it must hold for <u>all</u> operators $A \in R(O_2)$. So

$$\langle \hat{P}_1 \Omega, A \Omega \rangle = 0$$

for all $A \in R(O_2)$. But since $\{ A\Omega : A \in R(O_2) \}$ is dense, it follows that

$$\hat{P}_1 \Omega = 0 .$$

Hence, since Ω is cyclic for $R(O_1)$, $\hat{P}_1 = 0$, i.e.

$$P_1 = \langle \Omega, P_1 \Omega \rangle I .$$

So

$P_1 = 0$ or $P_1 = I$. But P_1 was arbitrary. Contradiction.

David

- Question: Measurement interaction U weakly local ($\exists\, O$: $R(O)$ unchanged)
  $\overset{?}{\Rightarrow}$ only local observable can be measured.
- Answer: No.

  Counter example: Take any observable in $R(O^\perp)$
  $A \in R(O^\perp)$ measured
  all observables B with $[B, A] = 0$ are unchanged.
  Now $[A, R(O)] = 0$
  $\Rightarrow$ ~~A∈R(O)~~ Observables in $R(O)$ unchanged
  in measurement of A.
  $\Rightarrow$ Measurement interaction localized outside O
  $\Rightarrow$ —, ——— weakly local.

  But: $O^\perp$ is not bounded $\Rightarrow$ there are observables Q in $R(O^\perp)$ which are not strictly local in the sense that
  $\exists V$ bounded open set of $\mathbb{R}^4$, s.t. $Q \in R(V)$.

(So probably the proposition 1 of Malament is really stronger than: $\langle \Omega | P | \Omega \rangle > 0$ $\forall P = P^2$ projectors in arbitrary $R(O)$ where O bounded, open $\subseteq \mathbb{R}^4$)

My own theorem is as follows:

The action of any element $A(0)$ of any local algebra on the vacuum state can never produce a state orthogonal to the vacuum state, provided the projector $P_{A(0)\Omega}$ is itself a member of the algebra.

Proof: Let A(0) be an element of R(0) for any 0. We require to show that $(\Omega, A(0)\Omega) \neq 0$, where Ω is the vacuum state, provided $P_{A(0)\Omega} \in R(0)$. Denote $A(0)\Omega$ by χ, and the projection operator onto χ by P_χ, assumed to be an element of R(0).

$$
\begin{aligned}
\text{Assume} \quad & (\Omega, \chi) &= 0 \\
\Rightarrow \quad & |(\Omega, \chi)|^2 &= 0 \\
\Rightarrow \quad & (\Omega, P_\chi \Omega) &= 0 \\
\Rightarrow \quad & (\Omega, P_\chi^2 \Omega) &= 0 \quad \text{since } P_\chi = P_\chi^2 \\
\Rightarrow \quad & (P_\chi \Omega, P_\chi \Omega) &= 0 \quad \text{since } P_\chi \text{ is self-adjoint} \\
\Rightarrow \quad & \| P_\chi \Omega \|^2 &= 0 \\
\Rightarrow \quad & P_\chi \Omega &= 0
\end{aligned}
$$

But it follows as an easy consequence of the Reeh-Schlieder theorem that Ω is a <u>separating vector</u> for any local algebra asssociated with a <u>bounded open set 0</u>.[4]

Hence we conclude from $P_\chi \Omega = 0$
that $P_\chi = 0$.
But this is impossible since it would imply $(\chi, P_\chi \chi) = 0$ whereas the value of this expectation value is clearly one. So by reductio the theorem is proved.

3 THE SIGNIFICANCE OF THE RESULT

Let us call $A(0)$ a superlocal observable if $P_{A(0)\Omega}$ is also local. Note that $\mathbb{1}$ although local is not superlocal, since P_Ω is not local, reflecting of course the fact that Ω is nonlocal. So, if A is superlocal, then $A' = A - \langle A \rangle_\Omega \mathbb{1}$ is not superlocal. Also $A'' = (1-P_\Omega)A$ is not even local let alone superlocal. Note that both A' and A'' have vanishing vacuum expectation values. But they provide no counter-example to our theorem which may be stated in the form that the vacuum expectation value of any superlocal observable is nonvanishing. One might wonder why the scalar field $\phi(x)$ in Klein-Gordon theory, which has the right micro-causality properties to be a candidate for a superlocal observable, is not a counter-example.

So our theorem says

$$\text{Prob}\ (\Omega \rightarrow \chi_{A(O)}) = |(\chi_{A(O)}, \Omega)|^2 \neq 0$$

or in other words there is a nonvanishing probability of finding the localized state $\chi_{A(O)}$ if we are in the vacuum state Ω , where $\chi_{A(O)}$ is as close as we like to <u>any</u> localized state of the field.

Notice that our theorem does not say Prob ($\Omega \rightarrow \chi) \neq 0$ for any state χ . This is clearly not true for the one-particle states, two-particle states, etc., which of course are all orthogonal to Ω . What our theorem shows is that superlocal operators can <u>never</u> produce pure many-particle states from the vacuum. There is always left behind a tail or trail of the vacuum state, which is what keeps Prob ($\Omega \rightarrow \chi_{A(O)}$) from being ever <u>exactly</u> zero.

Another way of putting this is that pure many-particle states in RQFT are themselves nonlocal entities which cannot be produced by superlocal operations acting on the vacuum.

Malament correctly divines the resolution of a counter-example to his theorem produced by the Unruh effect, in the circumstance that his result like mine, only applies to local operations - pure particle detectors have to be coupled to the field over <u>unbounded</u> regions of spacetime. So local detectors are not really detecting particles - they are firing alright, but in response to localized aspects of the field, not their global

aspects, which are connected to what may properly be called 'particle states'.

There are a couple of other points I want to make:

(1) Where does the energy come from to fire the detector? Essentially the detector has to feed energy <u>into</u> the field, exactly as happens with the detectors in the Unruh effect. It is a bit like detecting an electron in a region outside the nucleus of an atom where it ought to have negative kinetic energy. The observation feeds energy into the system so that any subsequent measurement of the kinetic energy would always be positive!

(2) We cannot exploit the vacuum correlations to convey information because they are just like the Bell correlations. As shown very clearly by Licht (1966), selective operations in O are required to produce arbitrary excitations in O′ – just hooking on non-selective measurement devices won't do the job.

4. CONCLUSION

Malament has given very elegant proofs of two theorems which highlight some features of the vacuum in RQFT, that could be thought paradoxical, but there are not really any paradoxes, just some remarkable physics.

Footnotes

1. See Reeh and Schlieder (1961).
2. See Fredenhagen (1985).
3. See Landau (1987).
4. See, for example, Streater and Wightman (1989). Theorem 4.3, p.139.

References

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